Refining Mathematical Meanings Through Multimodal Revoicing Interactions: The Case of "Faster"

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Author Note

This material is based on work supported by the National Science Foundation under Award No. 1321042.

Flood, V. J., Abrahamson, D. (2015, April). Refining mathematical meanings through multimodal revoicing interactions: The case of "faster." Paper presented at the annual meeting of the American Educational Research Association (Special Interest Group: Semiotics in Education: Signs, Meanings, Multimodality) Chicago, IL, April 16-20.

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Abstract

How do learners come to connect embodied experience with cultural-historical definitions? We compare two cases of a learner-instructor dyad negotiating the meaning of "faster" to investigate the role embodied, multimodal discourse plays in processes of collaborative semiosis within a technology enhanced discovery-based mathematics learning context (the Mathematical Imagery Trainer for Proportion). We implicate and characterize two new forms of multimodal revoicing interactions: (a) *Selective gestural repetition with co-timed elaborated verbal content* and (b) *Elaborated gestural content with co-timed repeated verbal content* in this process. Closer investigation of these forms may lead to deeper understanding of how responsive teaching supports embodied learning.

Keywords: Mathematics Learning, Embodied Interaction, Semiotics, Responsive Teaching, Conversation Analysis

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Objective

Learning scientists increasingly embrace the idea that mathematics learning, thinking, and knowing are embodied, multimodal processes. Following Ferrara (2014), we agree the question now is not *if* but *how.* The goal of this paper is to provide new insight into an enduring paradox concerning mathematics learning as an embodied phenomenon. How is it that learners come to organize the particularity of phenomenological experience (e.g., a feeling that something is "faster") with standardized cultural-historical mathematical formalisms (e.g., a mathematical definition of faster as more distance traveled per unit time)?

We implicate two new forms of *multimodal revoicing interactions* (Shein, 2012) as a means for supporting leaners' interleaving of their embodied experiences in the world with disciplinarily normative definitions of mathematics. This discursive mechanism for advancing semiosis is complementary to and builds from past accounts of mathematical ontogenesis via participation in social practice that includes problems, artifacts, and multimodal discourse (e.g., Radford, 2009).

Theoretical Framework

Revoicing involves *repeating* (re-stating information verbatim), *reformulating* (modifying content of original contribution), and/or *elaborating* (adding new information) students' contributions. O'Connors and Michaels (1993; 1996) first described the phenomenon, arguing it supported learner engagement in sophisticated academic classroom debates. Subsequent inquiries into revoicing in mathematics learning have shown the practice to be beneficial in supporting learners' in-depth exploration of mathematical ideas (e.g., Forman et al., 1998; Oh et al., 2008). However, past investigations focused on revoicing exclusively as a *verbal phenomenon*.

Instructors and learners jointly advance meanings moment-by-moment in face-to-face interaction using complex integrations of linguistic (e.g., spoken or written language), visual (e.g., images, three-dimensional models) and visuo-spatio-dynamic (e.g., action with artifacts, gesture) modes (Kress et al., 2001; Flood et al., 2015). Understanding the interactional processes by which learners and instructors negotiate and re-invent mathematical cultural forms together necessitates analytical attention to interactive bodily processes of objectification and signification that arise (Abrahamson, 2009; Arzarello et al., 2009; Nemirovsky & Ferrara, 2009; Radford, 2009; Roth & Thom, 2009; Wittmann, Flood, & Black, 2013). Thus, we argue that revealing the mechanisms by which meaning-making proceeds in revoicing interactions requires attention to the full range of embodied, multimodal semiotic resources (Goodwin, 2000) learners and instructors make available for each other.

Shein (2012) recently broadened the concept of revoicing to include nonverbal modalities in her investigation of a teacher's techniques to help English Language Learners recognize errors in geometric procedures. When students demonstrated erroneous procedures, they performed co-speech gestures over inscriptions. The teacher rehashed the mathematical procedure and repeated students' gestures while providing mathematically normative verbal descriptors. Prior to Shein's work, Arzarello and colleagues reported a similar phenomenon: mathematics teachers reproduce unconventional student signs (e.g., particular gestures) during their instructional explanations in order to link these signs with more conventional mathematical terms ("the semiotic game," e.g., Arzarello & Paola, 2007).

We characterize the type of multimodal revoicing Shein and Arzarello and colleagues describe as:

Form 1: Gestural repetition co-timed with elaborated verbal content

However, when learners communicate ideas through complex integrations of semiotic resources, there are a variety of potentially productive ways an instructor might reformulate (re-"voice") the multimodal contribution. We extend Shein and Arzarello's and colleagues work by identifying and characterizing two novel forms of multimodal revoicing. We add a second and third type:

Form 2: *Selective* gestural repetition with co-timed elaborated verbal content
Form 3: *Elaborated* gestural content with co-timed repeated verbal content

Form 2 is similar to Form 1 but involves an instructor who selectively repeats only *one gesture phrase* from a learner's more elaborate multi-phrased gesture and co-times this phrase with new verbal content. Form 3, on the other hand, can be thought of as the reverse of Form 1.

We present two cases that illustrate each of these qualitatively distinct forms of multimodal revoicing interactions. We show how these two forms of interaction provide learners with an opportunity to connect their embodied experiences to a more standardized cultural-historical mathematical definition of "faster."

Data Source and Analysis Method

Data Source:

Our episodes are drawn from a corpus of 23 video-recorded task-based interviews with a technology enabled embodied learning device: the Mathematical Imagery Trainer for Proportion (MIT-P). The MIT-P was designed to provide an interactive context to ground proportionality in sensorimotor schemes (Howison et al., 2011). Learners discover operatory schemes for achieving a non-symbolical goal state: make the computer screen green using hand-held remotes (Figure 1). Throughout the process, tutors support learners' efforts to signify their strategies mathematically by supplementing a series of symbolic artifacts (cursors, a Cartesian grid, numerals) on the screen.

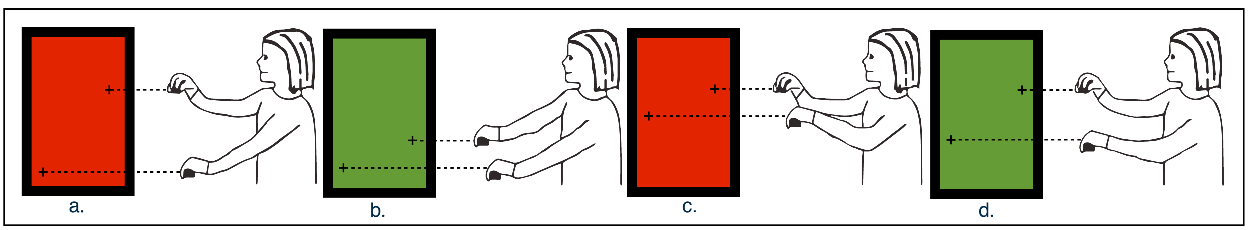

Figure 1. When the Mathematical Imagery Trainer for Proportion (MIT-P) is set to a 1:2 ratio, the right hand remote needs to be twice as high as the left hand remote in order to make the screen green. Learners receive green feedback when hand heights embody a 1:2 ratio (b, d) and red feedback when hand heights depart from this ratio (a, c).

During the interviews, learners and tutors negotiate descriptions for a variety of emergent mathematical phenomena. We selected the two episodes presented in this paper as exemplars to highlight the different features of the two new distinct forms of multimodal revoicing and because both dealt with the same theme: refining a definition of "faster."

Data Analysis:

Our multimodal microanalysis of video is a form of microethnographic analysis (Streeck & Mehus, 2005) inspired by conversation analysis. We use ELAN (Lausberg & Sloetjes, 2009) to spatio-temporally annotate video frame-by-frame to determine co-occurring segments of talk and gesture for each utterance-turn in revoicing interactions. Case studies (Yin, 2009) of micro-interactions afford rich, contextualized, and comprehensive inquiry into participants' methods of constructing meaning through which contributions to analytic generalizations may emerge.

Results

Selective Gestural Repetition With Co-Timed Elaborated Verbal Content (Form 2):

Our first vignette occurs during a discussion about speed with a pair of children—Lilah and Gideon—working with a concealed ratio setting of 1:2. Three tutors are present: Devon, Drake, and Joe. The children had already reported two strategies for performing manual transitions from each left-hand-and-right-hand "green" position to the next: either (1) ensure that the right hand (RH) is always *double as high* as the left hand (LH); or (2) move the hands with RH rising *double as fast* as LH. Drake asks the students whether they think there is any connection between these strategies (see Thompson, 1994).

Lilah offers an explanation for how the two strategies are connected. She explains that when the hands rise simultaneously and stop moving at a green position, RH must traverse a greater distance (it has to "lift higher") as compared to LH, and so RH must move faster. Her verbal description is accompanied by two different gestural phrases (Figures 2a & 2b).

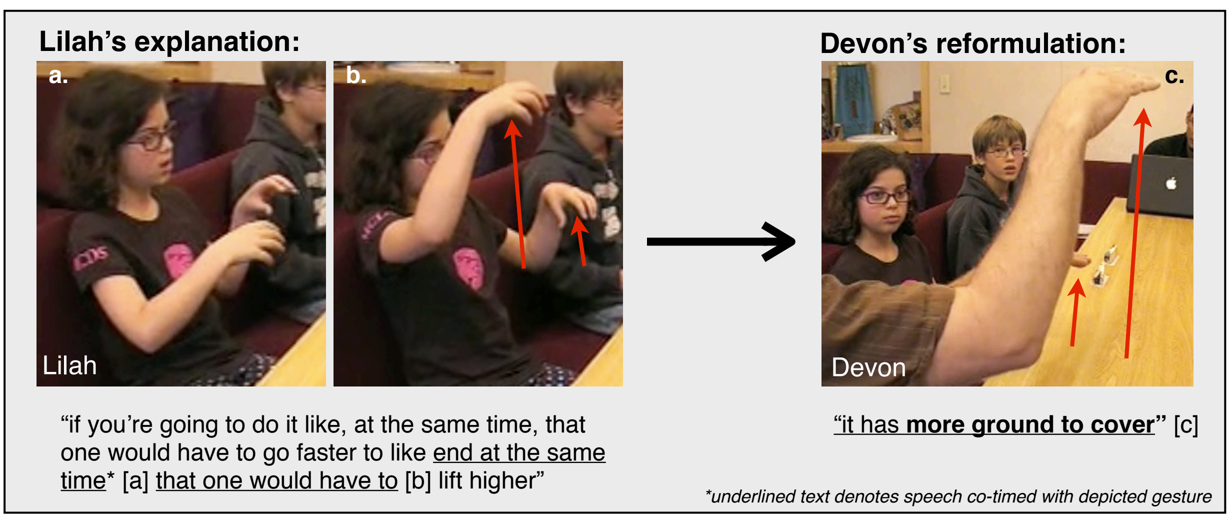

Figure 2. Lilah's explanation of "faster" has two distinct gestural phrases: first she holds both her hands at the same height (a) and then she performs a motion that would produce green feedback (b). Devon's revoicing turn reformulates Lilah's explanation by repeating only one of her gesture phrases (c) and co-timing it with a new verbal description.

Devon, in turn, signals an impending demonstration of his understanding of Lilah's suggestion (i.e. attribution, O'Connors & Michaels, 1993). He re-enacts Lilah's second gestural phrase—moving his RH and LH upward simultaneously with his RH traveling faster than his LH (Figure 2c). While producing this motion, Devon verbally *elaborates* Lilah's original description of one hand lifting higher by replacing it with a more universal, generalized description of spatial displacement: "more ground to cover" (Figure 2c).

Devon's multimodal revoicing builds selectively off of Lilah's available semiotic resources. Devon omits Lilah's ambiguous first gestural phrase. Producing green is never possible when the hands are at parallel heights. Therefore, conjoining "same time" [speech] and "same height" [gesture] is unclear. Further, Devon's verbal description modifies Lilah's to be less situated in the specific circumstances of the manual action of manipulating the device, moving towards a more universal, generalized definition of faster. The repeated gesture serves as a resource for Lilah to recognize the continuity between the two explanations: "more ground to cover" can be literally *seen* as equivalent to "lift higher." The result is a mathematically refined version of Lilah's original explanation: RH is going faster because it has more ground to cover than LH, in the same amount of time.

Lilah shows evidence for appreciating Devon's modification by skillfully incorporating his idea into her next turn: She concludes that to finish at the same time (and end up green, so that RH is double LH), the right hand has more ground to cover.

Elaborated Gestural Content With Co-Timed Repeated Verbal Content (Form 3):

In our second vignette, Amalia is working with the concealed ratio 2:3, with Devon and Drake. She shares the observation that RH is moving at a "higher speed" compared to LH. Devon invites her to elaborate. To describe "higher speed" further, Amalia implicates the differential spatial displacements required of LH and RH, respectively, each time she moves her hands to find a green position. She creates two different sized pinches with her hands and holds them up for Devon as she speaks (Figure 3a).

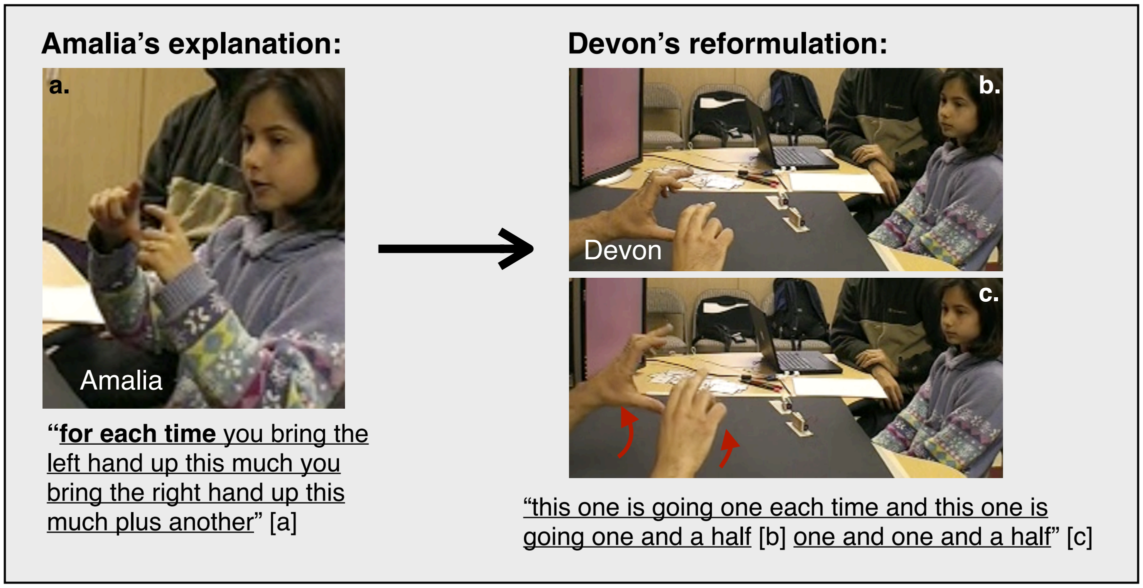

Figure 3. Amalia's description of "faster" consists of creating two different pinches, each with a different width to show spatial displacement (a). Devon's revoicing turn reformulates Amalia's idea by elaborating her gesture: he repeats it twice in quick succession (b and c).

Devon responds by attributing an idea to Amalia: "so you say the right hand goes faster." He goes on to explain by recognizably reproducing Amalia's gestural sign for the differential displacements by also fashioning his hands in two different sized pinch shapes. He repeats Amalia's original verbal construction but replaces Amaila's qualitative "this much" with two quantities. Devon also, however, *elaborates* Amalia's gesture by creating the two different sized pinches first at a low height (Figure 3b) and then raising both hands and recreating the two different sized pinches again (Figure 3c). This gestural elaboration introduces a way to parse time in the event of simultaneous asymmetrical spatial displacement—it shows the displacements occurring within two successive but equal periods of time.

Devon's iteration of Amalia's depiction of the differential spatial displacements introduces a new way of signifying that the differential displacements occur during equal spans of time, thus bringing Amalia's original explanation for "higher speed" closer to the normative definition of greater distance per unit time. The visual and verbal repetition of her construction of the two different displacements provides a resource for her to recognize how the gestured periods of time figure into her original explanation. Amalia subsequently endorses Devon's reformulation of her idea.

Conclusions

Lilah and Amalia both produce complex multimodal Gestalts (Mondada, 2014) that render elements of their felt physical experience of operating the device—a personal knowledge (Polanyi, 1958) of "faster"—available as nascent public signs. Through selective repetition, elaboration, or omission, contingent on the particular constitution of semiotic resources involved in each child's explanation, Devon shapes a more standardized, refined definition of "faster" from each child's contribution, resulting in the learner–tutor dyad's joint construction of a mathematical meaning that more closely resembles a mathematical version of "faster."

We have supplemented on the hitherto known form of multimodal revoicing (Arzarello Paola, 2007; Shein, 2012), which we name *"Form 1: Gestural repetition co-timed with elaborated verbal content,"* by characterizing two new forms: (2) *Selective gestural repetition with co-timed elaborated verbal content* and (3) *Elaborated gestural content with co-timed repeated verbal content* in this process. We propose that in both of these forms of multimodal revoicing the tutor's selective reproduction and elaboration of elements from learners' multimodal productions both (a) anchors reference (Goodwin, 2007) to a particular experience with cultural-historical forms (e.g., a mathematical definition of faster) and (b) publicly reifies a mathematical object in view of the learner.

We conjecture that there are likely many other productive forms of multimodal revoicing that warrant further investigation and characterization as part of better understanding practices of responsive teaching.

Significance

We add to a body of evidence (e.g., Shein, 2012) that instructors' selective reproduction and elaboration of learners' gestures during unfolding mathematical dialog is pedagogically effective, suggesting that gesture is a legitimate mode for making meaning in mathematics. Our work strives to legitimize—in educational research as well as practice—multiple ways of knowing and communicating about mathematics, an issue that is closely intertwined with the conference theme of "Toward Justice: Culture, Language, and Heritage in Education Research and Praxis." Currently, written, drawn, and spoken texts and diagrams are elevated as privileged systems for generating and conveying mathematical knowledge. Implicitly, multimodal revoicing suggests to students that gesture is a valuable and relevant way of knowing in the discipline.

We hope the tangible, concrete details from these exemplar cases of two new forms of multimodal revoicing interactions can serve as a resource and inspiration for practitioners to experiment and develop multimodal revoicing techniques as part of their own responsive praxis.

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